Asymmetrical Strobe Pulses for Multipath Mitigation in BOC GNSS Receivers

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Abstract—Binary offset carrier (BOC) modulations have been proposed for new Global Navigation Satellite System (GNSS) signals, since they achieve better tracking performance than phase-shift keying in the presence of channel noise and multipath. Besides, the concept of delay lock-loop based on BOC pulses can be extended to BO C modulations with significant advantage in the close-in multipath region. Herein, a design technique of asymmetrical strobe pulses for BOC receivers is proposed and analyzed. A target code discriminator response with desirable characteristics is defined allowing to determine the strobe pulse as the solution of an integral equation. The resulting pulse provides good multipath mitigation capability, extended code tracking range, and lack of false code-lock points.

I. INTRODUCTION

The concept of strobe correlator receiver for GNSS signals was considered by several authors [1]–[4]. It consists of correlating the inphase and quadrature components of the incoming signal with a locally generated spreading code and a gating signal. The gating signal is a train of specially tailored strobe pulses locked with the code sequence. By selecting the appropriate strobe pulse, different performances can be obtained in terms of code acquisition range, multipath mitigation capability, and robustness to thermal noise. Besides, the approach is powerful as most conventional delay lock loops (DLL), such as the early-late and the double-delta DLL, are particular cases of the strobe correlator, as shown, for instance, in [4].

The use of asymmetrical strobe pulses to reduce multipath errors was suggested in [2][3] for the GPS C/A signal. In [5], a new approach to the design of asymmetrical strobe pulses for BOC modulations, used in modernized GPS and in Galileo [6], was proposed.

The purpose of the present paper is to analyze the performance improvement that results from considering asymmetrical strobe pulses for the tracking of BOC(2n, n) waveforms in terms of multipath mitigation and robustness to thermal noise. The importance of this class of modulations stems from the use of BOC(10,5) in the modernized GPS military signal and in the Galileo system [7]. Following previous work [5][8], we start with a pre-defined (target) code discriminator response with suitable characteristics in terms of code tracking range and multipath mitigation capability and, taking into account the constraints, solve an integral equation whose solution is the desired strobe pulse. The resulting pulse provides extended code tracking range, good multipath mitigation capability, and lack of false code-lock points.

II. RECEIVER STRUCTURE

Henceforth, we restrict our attention to BOC sine-phased modulations. The baseband BOC(2n, n) signal is \( X(t) = S(t)C(t) \), where \( S(t) = \text{sign} \{ \sin(2\pi f_c t) \} \) is the BOC digital sub-carrier, with \( f_c = m f_0 \) and \( f_0 = 1.023 \text{ MHz} \), and \( C(t) = \pm 1 \) is the spreading code, with chip duration \( T_c = 1/(n f_0) \). The spreading code is defined by

\[
C(t) = \sum_k c_k w(t - k T_c),
\]

where \( c_k = \pm 1 \) is the chip corresponding to the interval \([k T_c, (k + 1) T_c]\) and \( w(t) = 1 \) in the interval \([0, T_c]\), being 0 otherwise.

As in [5][8]–[10], consider the overall structure of the frequency and code loops for the proposed receiver as sketched in Fig. 1. The inphase and quadrature components of the incoming signal, \( y_i(t) \) and \( y_q(t) \), are rotated by the locally generated phase, \( \tilde{\phi}(t) \). The resulting value \( Z(t) = z_i(t) + j z_q(t) \) is multiplied by the outputs of the BOC signal generator and the gating signal generator, respectively, \( X(t - \epsilon) \) and \( W(t - \epsilon) \), where \( \epsilon \) stands for the code delay error.

Define the complex cross-correlation between the generic signals \( a(t) \) and \( b(t) \), in the interval \([0, T]\), as

\[
R_{ab}(\epsilon) = \frac{1}{T} \int_0^T a(t) b^*(t - \epsilon) \, dt,
\]

where \( b^*(t) \) is the complex conjugate of \( b(t) \). Assume that \( h(t) \) is the impulse response of the baseband equivalent of the receiver’s front-end filter, then \( \tilde{X}(t) = X(t) * h(t) \) is the filtered version of \( X(t) \). The cross-correlation between the baseband input signal, \( Z(t) \), and the locally generated BOC signal, \( X(t - \epsilon) \), is given by

\[
R_{ZX}(\epsilon) = I_X(\epsilon) + j Q_X(\epsilon)
= aDR_{\tilde{X}Z}(\epsilon) \text{sinc}(\epsilon T) \exp(j \theta_\epsilon) + N_{IX} + j N_{QX},
\]

where \( A \) is the amplitude of the RF signal and \( D = \pm 1 \) is the navigation data. In (3), \( R_{\tilde{X}Z}(\epsilon) \) is the cross-correlation between \( \tilde{X}(t) \) and \( X(t) \), i.e. \( R_{\tilde{X}Z}(\epsilon) = R_{XZ}(\epsilon) * h(t) \), \( f_c \) is the frequency error, \( \theta_\epsilon \) is the phase error, and \( N_{IX} \) and \( N_{QX} \) are zero-mean independent Gaussian r.v. Their variances are \( \sigma_X^2 = N_0/2 \), where \( N_0/2 \) is the channel noise power spectral density.

Assume that \( W(t - \epsilon) \) is the locally generated gating signal, with

\[
W(t) = \sum_k c_k g(t - k T_c),
\]

and \( g(t) \) denotes the strobe pulse of width not exceeding the chip duration \( T_c \). The complex cross-correlation between the

![Fig. 1. Frequency and code loops in the proposed receiver.](image-url)
incoming signal and the gating waveform is given by
\[ R_{ZW}(\epsilon) = I_W(\epsilon) + j Q_W(\epsilon) = A D R_{\tilde{X}W}(\epsilon) \sin(f_c T) \exp(j \theta_\epsilon) + N_{IW} + j N_{QW}, \]
where \( R_{\tilde{X}W}(\epsilon) = R_{XW}(\epsilon) * h(\epsilon). \) In (5), \( N_{IW} \) and \( N_{QW} \) are zero-mean independent Gaussian r.v. with variance \( \sigma_i^2 = N_0 P_W / T, \) where \( P_W \) denotes the average power of \( W(\epsilon). \)

The code discriminator output of the proposed receiver is
determined by \[ d(\epsilon) = \Re\{R_{XW}(\epsilon) R_{\tilde{X}W}(\epsilon)\} \]

Neglecting the thermal noise contribution, the discriminator output is given by \( d(\epsilon) = A^2 R_{\tilde{X}W}(\epsilon) R_{\tilde{X}W}(\epsilon) \sin^2(f_c T). \) We will assume in the next sections, without loss of generality, that \( A = 1 \) and \( f_c = 0, \) yielding

\[ d(\epsilon) = R_{\tilde{X}W}(\epsilon) R_{\tilde{X}W}(\epsilon). \]

Henceforth, the baseband equivalent of the receiver’s front-end filter will be modeled as a fourth-order Butterworth lowpass filter, with bandwidth \( B \) (the corresponding 3 dB bandwidth of the RF filter is then \( 2B \)), and the filter is assumed to be phase-equalized so that the group delay is constant.

III. STROBE PULSE DESIGN

A desirable discriminator response should comply with the following design conditions: i) provide a good multipath mitigation capability, ii) maximize the code-lock range, and iii) minimize the existence of false code-lock points. The simplest multipath model assumes that the incoming signal is given by \( r(t) = s(t) + \alpha s(t-\tau) + n(t), \) where \( s(t) \) denotes the transmitted GNSS signal, \( \alpha \) is the relative amplitude of the reflected signal, \( \tau \) is the delay of the reflected signal regarding the line-of-sight signal, and \( n(t) \) is the thermal noise component. The multipath error envelope results from determining \( \epsilon \) from \( d(\epsilon, \tau) = 0 \) with \( n(t) \) neglected. In that case, the equation \( d(\epsilon, \tau) = 0 \) for the discriminator (6) is given by

\[ R_{\tilde{X}W}(\epsilon) R_{\tilde{X}W}(\epsilon) + \alpha \cos \varphi \times [R_{\tilde{X}W}(\epsilon) R_{\tilde{X}W}(\epsilon-\tau) + R_{\tilde{X}W}(\epsilon-\tau) R_{\tilde{X}W}(\epsilon)] + \alpha^2 R_{\tilde{X}W}(\epsilon-\tau) R_{\tilde{X}W}(\epsilon-\tau) = 0, \]

and the multipath error envelope is obtained by solving (8) for \( \epsilon \) with \( \cos \varphi = \pm 1, \) or equivalently

\[ R_{\tilde{X}W}(\epsilon) = \pm 0. \]

According to condition i), in the solution of (9), the effect of the term \( \alpha R_{\tilde{X}W}(\epsilon - \tau) \) should be minimized. Notice that the solution of \( R_{\tilde{X}W}(\epsilon) = 0 \) corresponds to the absence of multipath. Therefore, \( R_{\tilde{X}W}(\epsilon) \) or, due to (7), \( d(\epsilon) \) must be zero for \( \epsilon < 0. \) Condition ii) imposes that \( d(\epsilon) \) should be positive for \( \epsilon > 0 \) to achieve a good code acquisition range. Finally, condition iii) requires that the zero-crossings of \( d(\epsilon) \) must be avoided to eliminate the problem of false locks.

The BOC signal may be written as

\[ X(t) = \sum_k c_k q(t - k T_e), \]

where \( q(t) = \text{sign} \{ \sin(2 \pi f_s t) \}, 0 \leq t \leq T_e, \) is the BOC(\( m, n \)) pulse.

Using now (4) and (10) in (2), \( R_{XW}(\epsilon) \) is given by

\[ R_{XW}(\epsilon) = \frac{1}{T} \int_0^T \sum_k c_k c_l q(t - k T_e) g(t - l T_e - \epsilon) \, dt. \]

Assume that \( T \gg T_e \) and the chips \( c_k \) and \( c_l \) of a common sequence are independent and equally likely, then, when \( k = l, \) we obtain

\[ R_{XW}(\epsilon) = \frac{1}{T_e} \int_0^{T_e} q(\lambda) g(\lambda - \epsilon) \, d\lambda, \]

and the right member of (11) is statistically equal to zero for \( k \neq l. \)

The task is to solve the integral equation (12) to determine the strobe pulse \( g(t) \) when \( R_{XW}(\epsilon) \) is specified. The strobe pulse design was presented in [5] and a solution for BOC(\( m, n \)) was analyzed in [8]. Herein, we will find a solution for
BOC(2n, n) signals by defining a suitable target function \( R_{XW}(\epsilon) \). Note that the proposed solution is not unique: other target cross-correlation functions could be devised that verify the three design conditions.

Assume that the incoming signals have unbounded bandwidths, i.e., \( R_{XW}(\epsilon) = R_{X}(\epsilon) \) and \( R_{XW}(\epsilon) = R_{XW}(\epsilon) \) in (7). For BOC(2n, n) signals, the autocorrelation function, depicted in Fig. 2 (dotted line), is [11]

\[
R_X(\epsilon) = \begin{cases} 
1 - 7\epsilon / T_c, & |\epsilon| < T_c/4 \\
-2 + 5\epsilon / T_c, & T_c/4 \leq |\epsilon| < T_c/2 \\
2 - 3\epsilon / T_c, & T_c/2 \leq |\epsilon| < 3T_c/4 \\
-1 + |\epsilon| / T_c, & 3T_c/4 \leq |\epsilon| < T_c \\
0, & \text{otherwise},
\end{cases}
\]

and (12) may be written as

\[
R_{XW}(\epsilon) = \frac{1}{T_c} \sum_{k=0}^{3} (-1)^k \int_{kT_c/4 - \epsilon}^{(k+1)T_c/4 - \epsilon} g(t) \, dt.
\] (14)

The proposed target cross-correlation function is plotted in Fig. 3. According to the figure, it is required that \( S/T_c \leq 1/14 \approx 0.071 \). The desirable code discriminator response, given by (7), is sketched in Fig. 4 for several values of parameter \( S/T_c \). Applying the Leibniz’s rule to (14) leads to

\[
R'_{XW}(\epsilon) = \frac{1}{T_c} \left[ g(-\epsilon) - 2g(T_c/4 - \epsilon) + 2g(T_c/2 - \epsilon) - 2g(3T_c/4 - \epsilon) + g(T_c - \epsilon) \right]
\] (15)

from which we derive the following expression for \( g(t) \)

\[
g(t) = T_c R'_{XW}(-t) + 2g \left( t + \frac{T_c}{4} \right) + 2g \left( t + \frac{T_c}{2} \right) - 2g \left( t + \frac{3T_c}{4} \right) - g(t + T_c),
\] (16)

valid for \( 2S - T_c < t < 2S \), otherwise \( g(t) = 0 \). This allows to determine \( g(t) \) for positive values of \( t \) as \( g(t) = T_c R'_{XW}(-t) \) when \( t < T_c \). For negative values, the strobe pulse is recursively determined from (16). Fig. 5 shows the resulting pulses for two values of the parameter \( S/T_c \).

The code discriminator response versus the tracking error, \( \epsilon \), achieved with the proposed receiver is plotted in Fig. 6. Notice the agreement between these results and those depicted in Fig. 4. The anomalous shape for \( \epsilon > T_c/2 \) results from the impossibility to reconstruct the pre-defined function \( R_{XW}(\epsilon) \) with (14), for values of \( \epsilon \) larger than \( T_c/2 \), when the duration of pulse \( g(t) \) is constrained to be equal or less than \( T_c \).

IV. ANALYSIS IN THE PRESENCE OF NOISE

If the thermal noise contribution is considered, instead of (7), the code discriminator output of the proposed receiver is given by

\[
d(\epsilon) = R_{XX}(\epsilon) R_{WW}(\epsilon) + D \cos \theta_c [R_{XW}(\epsilon) N_{IX} + R_{XX}(\epsilon) N_{IW}] + D \sin \theta_c [R_{XW}(\epsilon) N_{QX} + R_{XX}(\epsilon) N_{QW}] + N_{IX} N_{IW} + N_{QX} N_{QW}.
\] (17)

Neglecting the contribution of the noise×noise terms, the r.v. \( d(\epsilon) \) is Gaussian with mean \( \bar{d}(\epsilon) = R_{XX}(\epsilon) R_{WW}(\epsilon) \) and variance

\[
\sigma_d^2 = \frac{N_0}{T} [R_{XW}(\epsilon) + R_{XX}(\epsilon) P_W].
\] (18)
Parameter $\gamma$ depends on the strobe pulse shape and the signal bandwidth through the quantities $S/T_c$ and $B T_c$, respectively, as shown in Fig. 9. Given the signal bandwidth, there is an optimal value for $S/T_c$ that minimizes the code tracking error variance. The optimal value of $S/T_c$ gets smaller when the bandwidth increases. Without band limitation, $\gamma = P_W / (S/T_c)^2$ and the tracking errors decrease monotonously when $S/T_c \to 0$ (dashdot line). For $S/T_c > 0.05$, the curve changes due to the interference between the rectangular pulses in $g(t)$, see Fig. 5.

V. Conclusion

In this paper we proposed an approach to the design of near-optimal asymmetrical strobe pulses for BOC(2n, n) GNSS receivers. We started with a target code discriminator response with suitable characteristics in terms of code tracking range and multipath mitigation capability, and solved an integral equation whose solution is the desired strobe pulse. This methodology allows more degrees of freedom in the choice of the strobe pulses and leads to the enhancement of the receiver performance achievable with symmetrical pulses in what concerns the code acquisition capability and false lock points elimination.

REFERENCES